SFERMIONS AND GAUGINOS IN A LORENTZ-VIOLATING THEORY

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In Lorentz-violating supergravity, sfermions have spin 1/2 and other unusual properties. If the dark matter consists of such particles, there is a natural explanation for the apparent absence of cusps and other small scale structure: The Lorentz-violating dark matter is cold because of the large particle mass, but still moves at nearly the speed of light. Although the R-parity of a sfermion, gaugino, or gravitino is +1 in the present theory, these particles have an "S-parity" which implies that the LSP is stable and that they are produced in pairs. On the other hand, they can be clearly distinguished from the superpartners of standard supersymmetry by their highly unconventional properties.

In Lorentz-violating supergravity[1,2], sfermions have spin 1/2. For one $SO(10)$ generation, there are 16 fields which are initially massless and right-handed. If half of these are converted to left-handed charge-conjugate fields[3,4], one obtains Lagrangian densities of the form

$$\mathcal{L}_R = \frac{1}{2}\left(-\bar{m}^{-1}\eta^{\mu\nu}\partial_\mu\psi_R^\dagger\partial_\nu\psi_R + \psi_R^\dagger i\sigma^\mu\partial_\mu\psi_R\right) + h.c. \tag{1}$$

$$\mathcal{L}_L = \frac{1}{2}\left(-\bar{m}^{-1}\eta^{\mu\nu}\partial_\mu\psi_L^\dagger\partial_\nu\psi_L - \psi_L^\dagger i\bar{\sigma}^\mu\partial_\mu\psi_L\right) + h.c. \tag{2}$$

where ψ_R and ψ_L are 2-component left-handed and right-handed fields, with $\bar{\sigma}^0 = \sigma^0 = \mathbf{1}$ and $\bar{\sigma}^k = -\sigma^k$ as usual, and $\eta^{\mu\nu} = diag(-1,1,1,1)$. (The spacetime coordinate system in which the initial fields are right-handed is defined above (9) of Ref. 3.) One can change from the dimension 3/2 fields ψ to conventional dimension 1 bosonic fields ϕ by absorbing a factor of $\bar{m}^{-1/2}$, but that would have no effect in the following arguments. The energy scale $\bar{m}$ is not determined by the present theory, but is assumed to lie well above 1 TeV– perhaps as high as 10^9 TeV or even the Planck scale.

Because of the minus sign in (2), one cannot couple left-and right-handed fields with a Dirac mass in the normal way. A Majorana mass is also not appropriate (except possibly in the case of sneutrinos). In the present paper we therefore consider a Lorentz-violating mass which is postulated to arise from supersymmetry breaking, at some high energy scale,

within the present Lorentz-violating theory. The Lagrangians then become

$$\mathcal{L}_R' = \bar{m}^{-1}\eta^{\mu\nu}\psi_R^\dagger\partial_\mu\partial_\nu\psi_R + \psi_R^\dagger i\sigma^\mu\partial_\mu\psi_R - m_R\psi_R^\dagger\psi_R \tag{3}$$

$$\mathcal{L}_L' = \bar{m}^{-1}\eta^{\mu\nu}\psi_L^\dagger\partial_\mu\partial_\nu\psi_L - \psi_L^\dagger i\bar{\sigma}^\mu\partial_\mu\psi_L - m_L\psi_L^\dagger\psi_L \tag{4}$$

where the prime indicates that a mass has been added and integrations by parts have been performed in the action $S = \int d^4x\,\mathcal{L}$. The mass term is invariant under a rotation, but not under a boost, since the transformation matrix for a boost is not unitary.

The equations of motion are

$$\bar{m}^{-1}\eta^{\mu\nu}\partial_\mu\partial_\nu\psi_R + i\sigma^\mu\partial_\mu\psi_R - m_R\psi_R = 0 \tag{5}$$

$$\bar{m}^{-1}\eta^{\mu\nu}\partial_\mu\partial_\nu\psi_L - i\bar{\sigma}^\mu\partial_\mu\psi_L - m_L\psi_L = 0. \tag{6}$$

As in Ref. 1, let each field be represented as

$$\psi = \sum_n a_n\psi_n \quad , \quad \psi_n(\vec{x}) = A_n\,\chi_n\,e^{i\vec{p}\cdot\vec{x}} \quad , \quad a_n(t) = e^{-i\omega_n t}a_n(0) \tag{7}$$

where $n \leftrightarrow \vec{p}, \lambda$ and there are two possibilities for the 2-component spinor χ_n:

$$\vec{p}\cdot\vec{\sigma}\,\chi_R = +|\vec{p}|\,\chi_R \quad , \quad \vec{p}\cdot\vec{\sigma}\,\chi_L = -|\vec{p}|\,\chi_L. \tag{8}$$

If we require only that each $a_n\psi_n$ satisfy the equation of motion for ψ, there are 4 solutions:

$$\pm 2\omega = -\bar{m} \pm \sqrt{\bar{m}^2 + 4p^2 \pm 4\bar{m}\,(p+m)} \tag{9}$$

where m represents either m_R or m_L and the three independent $\pm$ signs have the following meaning: The first $\pm$ sign, on the left side of the equation, is $+$ for a right-handed field ψ_R and $-$ for a left-handed field ψ_L. The last $\pm$ sign, under the radical, is $+$ for a right-handed solution containing χ_R and $-$ for a left-handed solution containing χ_L. Finally, the middle $\pm$ sign, preceding the radical, indicates that there are two solutions for a given ψ and χ, with the $+$ sign corresponding to the normal solution, for which $\omega \to 0$ as $|\vec{p}| \to 0$, and the $-$ sign to an extremely high energy solution, for which $|\omega| \to \bar{m}$ as $|\vec{p}| \to 0$.

We will now see that not all of the 4 above solutions are physical, because one of them may correspond to negative-norm states which are inadmissible in a proper positive-norm Hilbert space. After discarding any unphysical solution, however, we are still left with enough basis functions $\psi_n(\vec{x})$ to have a complete set of functions for (i) representing an arbitrary classical field

$\psi(\vec{x})$ and (ii) satisfying the quantization condition below. The canonical momenta are (in the notation of Ref. 1)

$$\pi_\psi^\dagger = \frac{\partial \mathcal{L}_\psi}{\partial \dot{\psi}} = \bar{m}^{-1}\dot{\psi}^\dagger \pm \frac{1}{2} i \psi^\dagger = \frac{1}{2} i \sum_n \left(\pm 1 + 2\omega_n / \bar{m}\right) a_n^\dagger \psi_n^\dagger \tag{10}$$

$$\pi_\psi = \frac{\partial \mathcal{L}_\psi}{\partial \dot{\psi}^\dagger} = \bar{m}^{-1}\dot{\psi} \pm \frac{1}{2} i \psi = \frac{1}{2} i \sum_n \left(\pm 1 - 2\omega_n / \bar{m}\right) a_n \psi_n \tag{11}$$

where the upper sign holds for a right-handed field and the lower for a left-handed field. We again quantize by interpreting ψ and $\pi^\dagger$ as operators, and requiring that

$$\left[\psi_\alpha\left(\vec{x}, t\right), \pi_{\psi\beta}^\dagger\left(\vec{x}\,', t\right)\right]_- = i\delta\left(\vec{x} - \vec{x}\,'\right)\delta_{\alpha\beta} \tag{12}$$

where α and β label the two components of ψ and $\pi_\psi^\dagger$. Following the same logic as in Ref. 1, one finds that this requirement can be satisfied if

$$2\bar{m}^{-1} \pm \omega_n^{-1} > 0. \tag{13}$$

Depending on the 3-momentum $\vec{p}$, there are either three or four solutions which meet this condition (and which are therefore included in the representation of the physical field operator ψ_R or ψ_L):

R	$\pm 2\omega = -\bar{m} + \sqrt{\bar{m}^2 + 4\left\vert\vec{p}\right\vert^2 + 4\bar{m}\left(\left\vert\vec{p}\right\vert + m\right)}$	all $\left\vert\vec{p}\right\vert$
R	$\pm 2\omega = -\bar{m} - \sqrt{\bar{m}^2 + 4\left\vert\vec{p}\right\vert^2 + 4\bar{m}\left(\left\vert\vec{p}\right\vert + m\right)}$	all $\left\vert\vec{p}\right\vert$
L	$\pm 2\omega = -\bar{m} + \sqrt{\bar{m}^2 + 4\left\vert\vec{p}\right\vert^2 - 4\bar{m}\left(\left\vert\vec{p}\right\vert - m\right)}$	$\left\vert\vec{p}\right\vert > P_+$ or $\left\vert\vec{p}\right\vert < P_-$
L	$\pm 2\omega = -\bar{m} - \sqrt{\bar{m}^2 + 4\left\vert\vec{p}\right\vert^2 - 4\bar{m}\left(\left\vert\vec{p}\right\vert - m\right)}$	all $\left\vert\vec{p}\right\vert$

where $2P_\pm = \bar{m} \pm \sqrt{\bar{m}^2 - 4\bar{m}m}$. Here the first column indicates whether the solution involves χ_R or χ_L, and again the upper and lower signs hold for ψ_R and ψ_L respectively. The same calculation as in (4.36) of Ref. 1 shows that the quantization condition above can be satisfied by choosing

$$A_n^* A_n = \left(\left\vert\omega_n\right\vert V\right)^{-1} \left\vert 2\bar{m}^{-1} \pm \omega_n^{-1}\right\vert^{-1} \tag{14}$$

in the case when there are 2 solutions for a given χ (either χ_R or χ_L) and

$$A_n^* A_n = 2\left(\left\vert\omega_n\right\vert V\right)^{-1} \left\vert 2\bar{m}^{-1} \pm \omega_n^{-1}\right\vert^{-1} \tag{15}$$

when there is only one solution.

As in Ref. 1 (and in standard physics), when the original energy ω is negative we reinterpret the destruction operator a for a particle as the

creation operator $b^\dagger$ for an antiparticle. If we now discard the extremely high energy solutions, and also restrict attention to momenta that are not extremely large, we have the following possibilities for both sfermions and antisfermions:

R	$\omega = \lvert\vec{p}\rvert + m$	all $\lvert\vec{p}\rvert$
L	$\omega = -\lvert\vec{p}\rvert + m$	$\lvert\vec{p}\rvert < m$

.

With a Lorentz-violating mass m, therefore, the group velocity $v = \partial\omega/\partial\,|\vec{p}|$ is 1 for right-handed sfermions (or antisfermions), in units with $\hbar = c = 1$, and -1 for left-handed sfermions. I.e., these Lorentz-violating particles have a highly Lorentz-violating energy-momentum relation, which implies that they travel essentially at the speed of light even though their masses are presumably comparable to 1 TeV. Furthermore, the velocity of a left-handed sfermion is antiparallel to its momentum, and there are no left-handed sfermion (or antisfermion) states for momenta with $|\vec{p}|\, c > mc^2$.

The present theory also contains gauginos etc. which can be candidates for the dark matter, but let us suppose here that the lightest supersymmetric partner is a neutral sfermion – i.e., a sneutrino– with the highly unconventional properties discussed immediately above. For simplicity, consider a circular orbit of radius r about a mass M. Let us restore c in the equations for clarity, and write $p = |\vec{p}|$. For $pc \ll mc^2$ the general formula for the centripetal force (with Newtonian gravity) implies that

$$pv/r = GMm/r^2 \quad \text{or} \quad r = R_S\left(mc^2/2pv\right) \tag{16}$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius. For nonrelativistic standard cold dark matter (CDM), the kinetic energy is $pv/2$, and for Lorentz-violating dark matter (LVDM) with $m \ll \bar{m}$ it is pv with $v \approx c$.[5] For a given kinetic energy, therefore, LVDM and CDM have orbits of comparable size in this simplistic picture. This can also be seen from the virial theorem[5] $\langle pv\rangle = \langle \vec{p}\cdot\vec{v}\rangle = -\left\langle \vec{F}\cdot\vec{r}\right\rangle = \langle GM\,(r)\,m/r\rangle$ which implies that the kinetic energy is equal in magnitude to the gravitational potential energy for LVDM, and to one-half the potential energy for CDM, so dark matter with a given distribution of kinetic energies will have comparable large-scale trajectories in both models. On the other hand, the binding energy is vastly smaller for LVDM, and this leads to the hope that LVDM can provide a natural explanation for the apparent discrepancy between observations and CDM simulations in regard to cusps and other small scale structure. There is some tentative confirmation of this idea in preliminary computer simulations[5].

In Lorentz-violating supergravity[2], at energies low compared to $\bar{m}$, the fermion fields Ψ_f and sfermion fields Ψ_b are coupled in the following way to the gauge fields A^i_μ, the gaugino fields $\tilde{A}^i_\mu$, the gravitational vierbein e^μ_α, and the gravitino field $\tilde{e}^\mu_\alpha$:

$$S_L = \int d^4x\, e\, \Psi^\dagger\, iE^\mu_\alpha \sigma^\alpha \mathcal{D}_\mu\, \Psi \quad , \quad \mathcal{D}_\mu = \partial_\mu - i\mathcal{A}^i_\mu t_i \tag{17}$$

$$\Psi = \begin{pmatrix} \Psi_b \\ \Psi_f \end{pmatrix} \quad , \quad \mathcal{A}^i_\mu = \begin{pmatrix} A^i_\mu & \tilde{A}^{i\dagger}_\mu \\ \tilde{A}^i_\mu & A^i_\mu \end{pmatrix} \quad , \quad E^{\ \mu}_{\ \alpha} = \begin{pmatrix} e^\mu_\alpha & \tilde{e}^{\mu\dagger}_\alpha \\ \tilde{e}^\mu_\alpha & e^\mu_\alpha \end{pmatrix} . \tag{18}$$

We have generalized the usual vocabulary in a natural way, so that the superpartner of the graviton is defined to be the gravitino, and the superpartners of gauge bosons to be gauginos, even though these fermions have quite unconventional properties. In particular, each superpartner has both the same quantum numbers and the same spin as the Standard Model particle. This means that the conventional R-parity is +1 for sfermions, gauginos, and gravitinos. However, since each vertex involves an even number of supersymmetric partners we have conservation of an "S-parity", which is -1 for sfermions, gauginos, and gravitinos, and +1 for their Standard Model counterparts. (This is also true when the kinetic terms for the force fields are included, as will be discussed elsewhere.) Then the lightest supersymmetric partner (LSP) cannot spontaneously decay into lighter Standard Model particles. For the same reason, sparticles are always created in pairs. Parton-parton or lepton-lepton interactions will lead to production of an even number of supersymmetric partners, just as in standard supersymmetry with R-parity conservation. However, the sparticles predicted by the present theory are clearly distinguished by their highly unconventional properties.

References

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